Demand Estimation with Market-Share Rankings

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Rankings instead of Market-Shares

- Platforms often publish a ranking of most-to-least sold products instead of sales
  - E-commerce platforms: Amazon BSR, Ebay top selling products, Walmart and others
  - Website visits: Alexa ranking
  - App stores: IOS, Google play top downloaded apps

- Demand estimation requires exact market shares:
  - Usual solution: estimate a power law that links rankings to sales
  - Impute missing market shares
    - Assumes sales are ranked according to power law [not always true]
    - Noisy when only a few product sales' are available

- This practice is **very common**
  - 400+ papers use rank to sales imputation (Liebowitz and Zentner, 2024)
  - We show that demand estimates are very sensitive to the specific power law that's used

This Paper: Estimate Demand from Rankings

Goal: Estimate demand systems with market-level data when market shares are unobserved but rankings of product sales are available

Current common approach:

- Select a single vector of market shares consistent with the ranking
- Perform standard demand estimation assuming selected shares are exact

This paper:

- Define data restrictions implied by the ranking
- Map all feasible share vectors into demand utilities
- Use moment inequalities to obtain bounds on demand parameters and welfare

This paper: Preview of the Results

Contributions:

- Characterize sharp identified set for demand parameters given ranking
- Find bounds of identified set via convex optimization $\rightarrow$ computationally feasible

Features:

- Easy to incorporate observed shares
- Allow instrumental variables
- Cover standard differentiated demand systems (logit, probit, random coefficients)

Literature

- Power laws and rankings Chevalier and Goolsbee (2003), Reimers and Waldfogel (2021), Gutierrez (2022), Yu (2024), Gortmaker (2025)

  $\rightarrow$ relax dgp assumption and apply partial identification

- Moment inequalities for revealed preferences Ho and Rosen (2017), Kline, Pakes and Tamer (2021), Dickstein and Morales (2018), Dickstein, Jeon and Morales (2025), Porcher, Morales and Fujiwara (2025)

  $\rightarrow$ maintain structural assumptions but uncertainty over realized quantities

- Interval regression and inference Manski and Tamer (2002), Beresteanu, Molchanov, and Molinari (2011), Chernozhukov, Chetverikov, and Kato (2019)

  $\rightarrow$ extend interval regression framework and apply standard inference procedures

Notation Overview

- Feasible Shares: $\mathcal{S}$
  - All market-share vectors that could justify the observed ranking
  - Defined by linear inequalities

- Feasible Utilities $\mathcal{D}$
  - All vectors of product utilities that can be mapped to feasible shares
  - Non-convex set $\rightarrow$ computationally intractable

- Convex hull of feasible utilities set: $\text{co}(\mathcal{D})$
  - Computationally tractable
  - Use it to obtain linear moment inequalities for demand parameters

- Demand Parameters Θ_I
  - Identified set of demand parameters
  - Estimate counterfactuals and bound welfare effects

Ranking Data Restrictions

- Data restrictions on market shares implied by ranking:
  1. All shares must sum to one $s_0 + \sum_j s_j = 1$
  2. Monotonically decreasing shares in the ranking $s_j \geq s_k \iff r_j < r_k$

- Together they define a simplex of feasible market shares:

$$\mathcal{S} = \left\{ s \in (0,1)^J : s_j \geq s_k \iff r_j < r_k, \sum_j s_j = 1 - s_0 \right\}$$

- Assume:
  - Outside share is always observed s_0 [can be relaxed]
  - Minimum market share $\epsilon > 0$

Model: Discrete-choice Framework

- Agent i purchases product j iff: $u_{ij} \geq u_{ij'} \; \forall j' \neq j$

- Utility is additively separable in three elements:

$$u_{ij} = \delta(x_j, \xi_j) + \nu_i(x_j) + \varepsilon_{ij}$$

  - $\delta_j = x_j'\beta + \xi_j$: systematic utility level (equal for all consumers)
  - ν_{ij}: random-coefficient
  - ε_{ij}: individual-specific taste shock

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- If $\varepsilon \sim$ T1EV

$$s_j = \int_\nu \frac{\exp(\delta_j(x_j, \xi_j) + \nu_i(x_j))}{1 + \sum_j \exp(\delta_j(x_j, \xi_j) + \nu_i(x_j))} dF(\nu_{ij})$$

Identification: Point and Set

- Given distribution of ν, can invert each vector s into corresponding δ

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  1. Structural inversion to recover mean utilities from shares $\delta = \sigma^{-1}(s)$
  2. Linear moment: $\mathbb{E}(\xi|Z) = 0$ to point identify demand parameters θ

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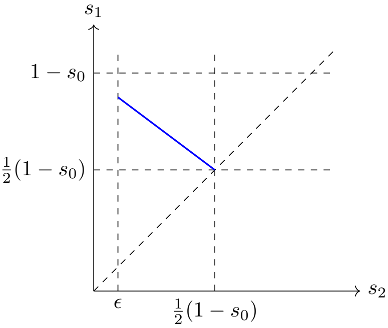
- If multiple shares are possible:
  - Structural inversion implies set of mean utilities $\mathcal{D} = \sigma^{-1}(\mathcal{S})$
  - Linear Moment $\mathbb{E}(\xi|Z) = 0$ to set identify parameters Θ

Building Moment Inequalities

One market, two goods, simple logit:

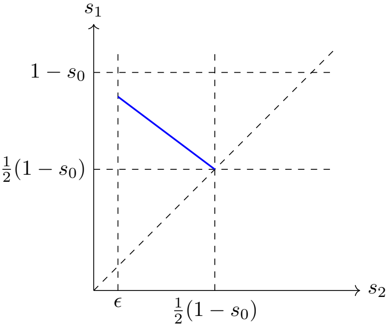


(a) Shares $\mathcal{S}$

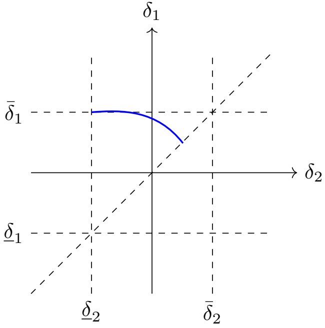
$\mathcal{S}$ is defined by a system linear restrictions

Building Moment Inequalities

One market, two goods, simple logit:



(a) Shares $\mathcal{S}$



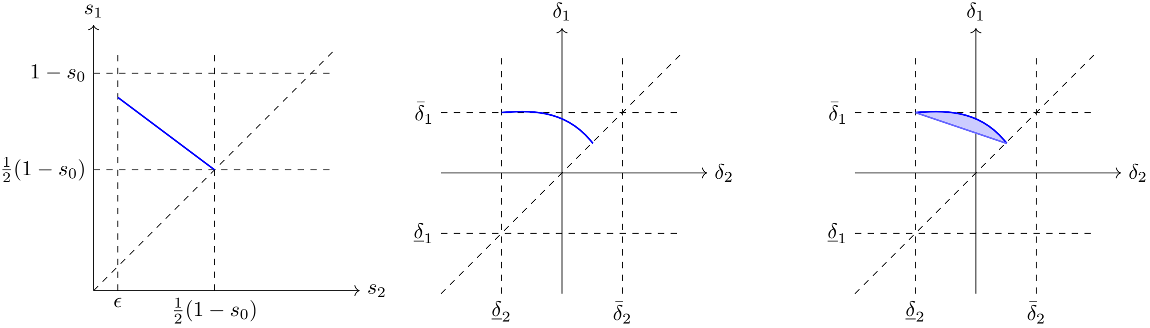
(b) Systematic Utilities $\mathcal{D}$

$\mathcal{D}$ contains all systematic utilities vectors that can be inverted to elements of $\mathcal{S}$

- Not convex $\implies$ cannot apply moment inequalities to find Θ

Building Moment Inequalities

One market, two goods, simple logit:



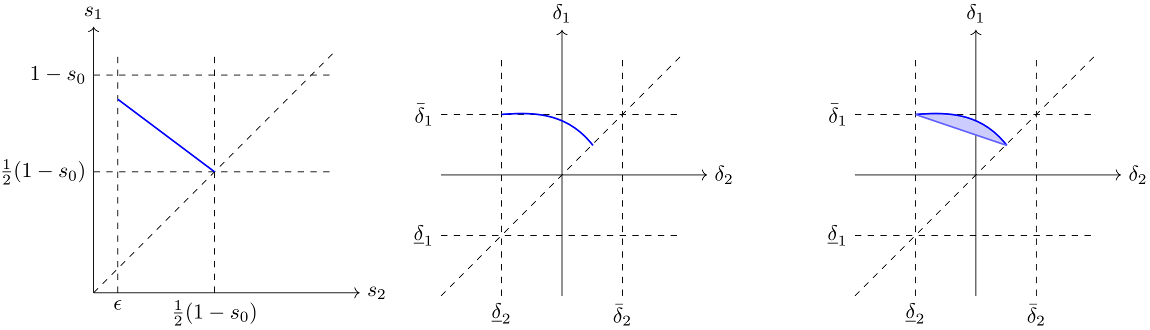
(a) Shares $\mathcal{S}$ (b) Systematic Utilities $\mathcal{D}$ (c) Convex Hull co($\mathcal{D}$)

co($\mathcal{D}$) can be expressed using only linear inequalities

- Convex set $\implies$ can apply moment inequalities to find Θ

Building Moment Inequalities

One market, two goods, simple logit:



(a) Shares $\mathcal{S}$ (b) Systematic Utilities $\mathcal{D}$ (c) Convex Hull co($\mathcal{D}$)

Lemma: we can go from $\mathcal{D} \to$ co($\mathcal{D}$) without losing sharpness intuition

Building Moment Inequalities

- By definition $\text{co}(\mathcal{D})$ is obtained via linear restrictions:
  - $\text{co}(\mathcal{D}) = \left\{ \delta : v'\delta \leq \sup_{d \in \mathcal{D}} v'd \quad \forall v \in \mathbb{S}^{J-1} \right\}$ with $\mathbb{S}^{J-1}$ unit sphere

- Moment Inequalities:
  - $\delta_j = x_j'\beta + \xi_j$
  - Moment condition: $\mathbb{E}[\xi|Z] = 0$
  - Identified set:

$$\Theta_0 = \left\{ b \quad \middle| \quad \mathbb{E}\left[v'Xb \middle| Z \right] \leq \sup_{d \in \mathcal{D}} v'd, \quad \forall v \in \mathbb{S}^{J-1} \right\}$$

- Two-step procedure:
  - Find $\sup_{d \in \mathcal{D}} v'd$
  - Apply linear moment inequalities to find β

Calculating Inequalities Bounds

Propose two conditions to feasibly compute $\sup_{d \in \mathcal{D}} v'd$

- **A1)** Products are weak substitutes
- **A2)** Limited heterogeneity in Preferences
  - Logit, nested logit, and standard random coefficients allowed (normal, lognormal)
  - Rule out bimodal heterogeneity in preferences

Results

1. Only need to evaluate positive and negative directions v
2. Positive limits obtained via convex optimization program
3. Negative limits obtained enumerating vertices

lemma computation

Overview of the Procedure

0. Draw finite number of directions $v \in \mathbb{S}^{J-1,+}$
   - Number of directions need not be large, just chosen correctly subvector

1. Obtain upper and lower bound of $co(\mathcal{D})$ for every direction v

2. Conduct inference via standard moment inequality approaches subvector
   - For simplicity we use Chernozhukov, Chetverikov, and Kato (2019)
   - Any method that allows for many moment inequalities works

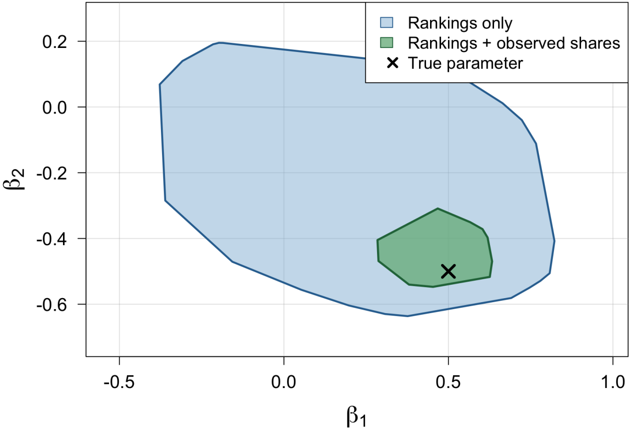
3. Evaluate counterfactual
   - Merger simulation
   - Adding/removing products

Note: for BLP the procedure is the same, just requires outer grid over nonlinear parameter

Logit IV Simulation: Identified Set Shrinks with Observed Shares

$$u_{ijt} = \beta_1 x_{1jt} + \beta_2 x_{2jt} + \xi_{jt} + \varepsilon_{ijt}$$

- In blue: only ranking, $J = 20, T \to \infty$
- In green: 5 observed shares + ranking



Coverage of the True Coefficient

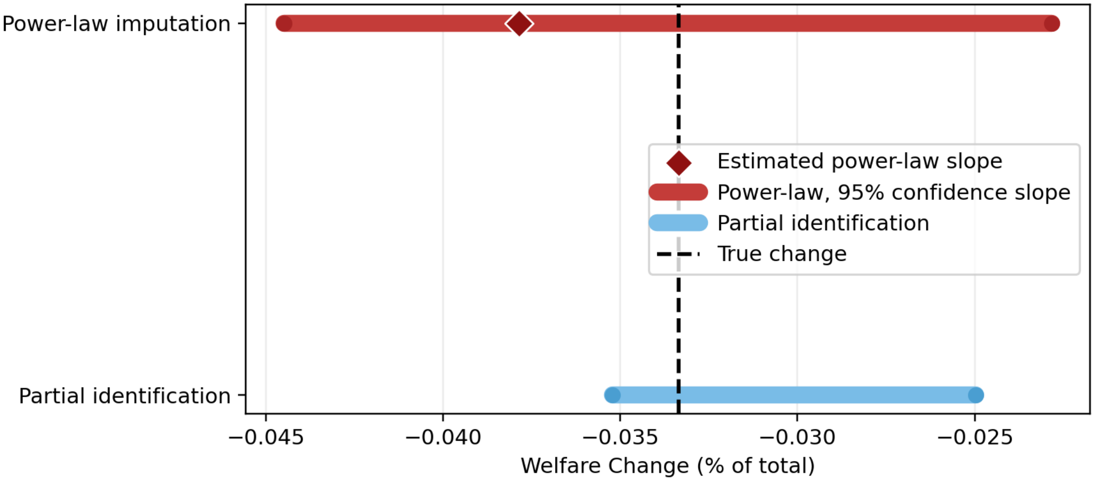
$$u_{ijt} = \beta_1 x_{1jt} + \beta_2 x_{2jt} + \xi_{jt} + \varepsilon_{ijt}$$

	Confidence set coverage of true β		
Products (J)	Power-law imputation	Partial identification	Max. distance to CS boundary
5	0.99	1.00	0.335
10	0.78	1.00	0.397
20	0.75	1.00	0.407
30	0.71	1.00	0.426
40	0.65	1.00	0.429
50	0.56	1.00	0.434

J = 20, T = 500, 5 observed shares, confidence set coverage across 2000 simulations

power-law imputation formula

Merger-Welfare Simulation: Results

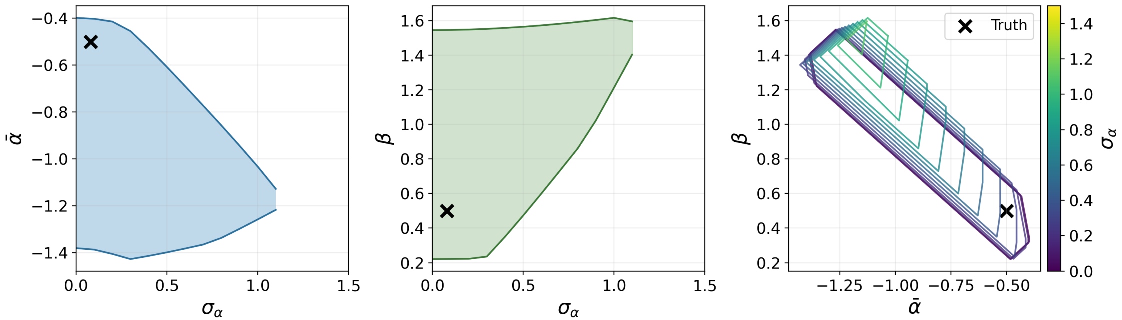


$J = 20$, $T = 100$, 5 product shares observed, merger between products 3 and 4

Pseudo Code

BLP Simulation: Identified Set on a Grid of Sigma

$$u_{ijt} = -\alpha_i p_{jt} + \beta x_{jt} + \xi_{jt} + \varepsilon_{ijt} \quad \alpha_i \sim \mathcal{N}(\bar{\alpha}, \sigma_\alpha^2)$$



Conclusions

- Construct a new method to estimate demand when market shares are unavailable but their ranking is observed
- Turn nonlinear demand inversion with incomplete shares into tractable moment inequalities
- Provide conditions for a well-defined confidence set and compare it with power-law imputation
- Accommodate endogenous prices and random coefficients
- Bound economically relevant counterfactuals, including merger welfare effects

Power-Law Imputation

Estimate the power law on the observed rank–quantity pairs:

$$\log q_{jt} = \gamma + \eta \log r_{jt} + \varepsilon_{jt}$$

For the fitted slope $\widehat{\eta}$, or any sensitivity-grid value η, form power-law weights:

$$\widehat{q}_{jt}(\eta) = \exp(\widehat{\gamma}) r_{jt}^{\eta}.$$

Project these weights onto the same feasible set used by our method:

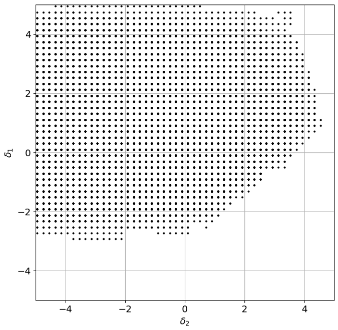
$$\widehat{s}_t(\eta) = \arg\max_{s \in \mathcal{S}_t} \sum_{j=1}^{J} \widehat{q}_{jt}(\eta) \log s_{jt}.$$

Here $\mathcal{S}_t$ preserves observed shares, the ranking, and $\sum_j s_{jt} = 1 - s_{0t}$. The intercept cancels in the projection. The resulting $\widehat{s}_{jt}(\eta)$ are treated as exact shares.

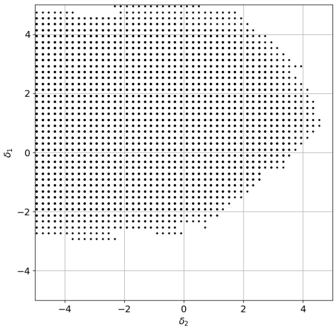
Draw Directions: Subvector Identification 1

Evaluating co($\mathcal{D}$) still requires exponentially many directions

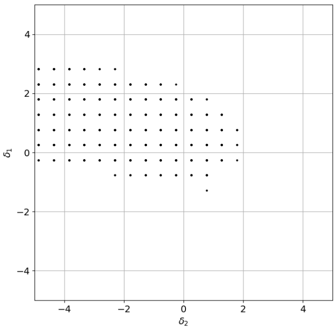
- With just $J = 5$ we easily run into the curse of dimensionality



(a) 50 Directions



(b) 100 Directions



(c) 1000 Directions

Draw Directions: Subvector Identification 2

Issue:

- We are trying to span all directions in J-dimensional space
- Effectively we only need to bound $\beta \in R^K$ demand parameters

Solution:

1. Draw $\tilde{v} \in \mathbb{S}^K$
2. Obtain market-specific directions $v_{jt} = \max\{0, x'_{jt}\tilde{v}_j\}$

In our simulations 30 directions are enough for $K = 2$ and $J = 100$

Sharpness of the set

Lemma 1

Aumann Expectations over $\mathcal{D}$ and $co(\mathcal{D})$ are equivalent

$$\mathbb{E}_A[\delta \in \mathcal{D}|Z] = \mathbb{E}_A[\delta \in co(\mathcal{D})|Z]$$

Intuition:

- Take $\mathcal{D}_t = [\delta_1, \delta_2]$
- Unobserved selection rule chooses between δ_1, δ_2
- Taking expectation over unobserved selection rule is equivalent to:

$$\delta = \lambda\delta_1 + (1-\lambda)\delta_2 \quad \forall\lambda \in [0,1]$$

No information is lost from $\mathcal{D}$ to co($\mathcal{D}$)

back

Removing Redundant Inequalities

- Evaluating $\sup_{d \in \mathcal{D}} v'd$ is still computationally difficult
  - Requires evaluating d in non-convex set $\mathcal{D}$
  - Often no analytical representation of the set $\mathcal{D}$ (for example BLP)

- We provide two conditions to reduce the problem to a convex optimization
  - **A1) Connected Substitutes**: products are weakly substitutable and there is a strict substitution path between any two products. (Berry, Gandhi, and Haile, 2015)
  - **A2) Limited Heterogeneity in Preferences**: the hessian of the inverse demand function is negative semi-definite for every good j and for every $s \in \text{int}(\mathcal{S})$
    $\rightarrow$ Normally distributed random coefficients ok
    $\rightarrow$ Rules-out bimodal heterogeneity

- **Conjecture**: Under (A1), (A2) graph($\mathcal{D}$) is weakly decreasing and concave:
  - We can just evaluate positive and negative directions v to define the convex hull

back

Finding Bounds

- Upper bounds via convex program:

$$\sup_{d\in\mathcal{D}} v'd \quad \forall v \in \mathbb{S}^{J-1,+}$$

  - Logit:
    For every direction v solve convex program

$$\max_{s\in\mathcal{S}} \sum_j v_j(\log(s_j) - log(s_0))$$

- Lower Bounds via evaluation of vertices
  - Since $\mathcal{S}$ is a simplex, minimum must be at a vertex
  - Need to evaluate only J points for every direction v

back

Identifying co($\mathcal{D}$) when J is large

When J is high, drawing $co(\mathcal{D})$ becomes computationally infeasible:

- Every additional product in the market adds a new dimension to the convex set

However we don't care about δ per se, we care about the parameters β and ξ:

- Large literature on subvector identification
- We can find efficient directions via SVD factorization of X
- Practically, draw u in a way that spans most of the variation in X:

$$v_{lt} = u_l X_t$$

back

Conditional Moments to Quantile Functions

- Conditional Expectations:

$$\mathbb{E}_t\left(v'Xb \leq \sup_{d\in\mathcal{D}} v'd \middle| Z\right) \quad \forall v \in \mathcal{V}$$

  - $\mathcal{V} = \{v \in \mathbb{S}^{J-1,+}\} \cup \{-v \in \mathbb{S}^{J-1,+}\}$
- Transform into unconditional via quantile functions $g(.) \in \mathcal{G}$ and evaluate null hypothesis for every b:

$$H_0(b): \quad \mathbb{E}\left((v'Xb - \sup_{d\in\mathcal{D}} v'd)g(Z)\right) \leq 0 \quad \forall g \in \mathcal{G}, v \in \mathcal{V}$$

  - Use Chernozhukov et al. (2019) for the evaluation of H_0

back

Merger-Welfare Simulation: Design

Simulated Data

- 20 products, 100 markets
- Top 5 products observed, rest only ranking
- Products 3 and 4 merge

Estimation and Counterfactual

- Fix a grid of parameters: $\{\theta_g\}_{g=1}^{G} \subset \Theta_I$.
- For every θ_g:
  - Solve the BLP inversion on a fixed, market-specific grid $\{s_{t\ell}\}_{\ell=1}^{L} \subset \mathcal{S}_t$ and obtain $\delta_{t\ell}(\theta_g)$.
  - Calculate the merger-welfare change $\Delta W_{t\ell}(\theta_g)$ for every candidate point.
  - Solve for min and max of ΔW with a linear program that reweights the candidate points subject to the moment restrictions.
  - Obtain $[\underline{\Delta W}(\theta_g), \overline{\Delta W}(\theta_g)]$.
- Identified set of welfare changes:

$$\Delta \mathcal{W} = \bigcup_{g=1}^{G} [\underline{\Delta W}(\theta_g), \overline{\Delta W}(\theta_g)].$$

back